20/8/2020 B.Sc. Part II 4th Paper

Dynamics

Q: Prove that if the tangential and normal accelerations of a particle describing a plane curve be constant throughout the motion, the angle ψ which the direction of motion turns in time t, is given by $\psi = A \log(1+Bt)$.

Soln. Given that

tangential acceleration $= \dfrac{d^2s}{dt^2} = v\dfrac{dv}{ds} = \text{Constant}$

and normal acceleration $= \dfrac{v^2}{\rho} = \text{Constant}$

Let $\dfrac{d^2s}{dt^2} = \text{Constant} = k$ (say)

$\cancel{\Rightarrow v\dfrac{dv}{ds} = k \Rightarrow v\,dv = k\,ds}$

$\Rightarrow \dfrac{d^2s}{dt^2} = k \Rightarrow \dfrac{d}{dt}\left(\dfrac{ds}{dt}\right) = k$

Integrating with respect to t, we get

$\Rightarrow \dfrac{ds}{dt} = kt + k_1$

$\Rightarrow v = kt + k_1$ —— (1)

$\because \dfrac{v^2}{\rho} = \text{constant}$

$\Rightarrow \dfrac{(kt+k_1)^2}{\rho} = c$

$\Rightarrow (kt+k_1)^2 = c\rho = c \cdot \dfrac{ds}{d\psi} \quad \left[\because \rho = \dfrac{ds}{d\psi}\right]$

$\Rightarrow (kt+k_1)^2 = c \cdot \dfrac{ds}{dt} \cdot \dfrac{dt}{d\psi} = c \cdot (kt+k_1) \dfrac{dt}{d\psi}$

$\Rightarrow kt + k_1 = c \dfrac{dt}{d\psi}$

$\Rightarrow \dfrac{c\,dt}{kt+k_1} = d\psi$. integrating, we get

$\Rightarrow c \displaystyle\int \dfrac{dt}{kt+k_1} = \int d\psi$

$\Rightarrow \dfrac{c}{k} \log(kt+k_1) = \psi + c_1$ —— (2)

Let $\psi = 0$ when $t = 0$

So (2) $\Rightarrow \dfrac{c}{k} \log k_1 = c_1$.

Putting this value in (2), we get

$\dfrac{c}{k} \log(kt+k_1) = \psi + \dfrac{c}{k} \log k_1$

$\Rightarrow \psi = \dfrac{c}{k} \log\left(\dfrac{kt+k_1}{k_1}\right)$

$\Rightarrow \psi = \dfrac{c}{k} \log\left(1 + \dfrac{k}{k_1} t\right)$

$\Rightarrow$ Let $\dfrac{c}{k} = A$, $\dfrac{k}{k_1} = B$

$\Rightarrow \psi = A \log(1 + Bt)$